

APPENDIX F. EXAMPLE COMPUTATIONAL TECHNIQUE VALIDATIONS

This appendix provides more detailed descriptions and example implementation of the required elements for computational technique validation as described in sections 5.8.4 and 5.8.5, i.e.,

- (a) the selection and description of the critical experiments used in the validation, or an appropriate reference that describes the experiments in adequate detail to permit reconstruction of computational input,
- (b) the selection and description of the computational method that is to be validated along with any necessary data for performing calculations or comparisons (e.g., neutron cross sections, material bucklings, limiting surface densities, or other similar data),
- (c) the selection and description of the computer/calculator platform and associated operating system used in the validation,
- (d) the nuclear properties, such as cross sections, that should be consistent with experimental measurements of these properties,
- (e) a description of similarities and differences between the critical experiments and the calculational models used for the validation,
- (f) all geometric, material, and nuclear physics related input variables used for the validation of the calculational or comparative method, with sketches provided,
- (g) the basis for the calculational or comparative bias and the determination of an acceptance criterion for calculated subcritical results, and
- (h) establishing the areas of applicability of the calculational or comparative bias and the acceptance criterion developed from the validation effort.

F.1 Selection and description of critical experiments. The selection and description of critical experiments used as benchmarks for the calculational method validation should be similar to, and representative of, the problems that are to be evaluated. The benchmarks' physical compositions, geometric configurations, and other nuclear characteristics should be reviewed to ensure applicability (similarity) to the future problems for which the validation is intended. Unfortunately, critical experiments available for benchmarking tend to emulate single units. A particular problem evaluation may require calculations for a single unit, as well as arrays of units (such as in fissionable material package or storage array evaluations). Such a problem poses a difficulty in benchmark selection because there is a paucity of critical experiments of large arrays. Because of these concerns, it may be necessary to model a wide variety of benchmark experiments to adequately assess the validity of the calculational method used in the evaluation. Sufficient numbers and quality of experiments shall be selected to provide a statistically justifiable basis for subcritical acceptance criteria.

F.2 Selection and description of the computational method. The selection of the computational method should be related to the particular expertise and experience of the criticality safety specialist performing the validation and should be related to the difficulty of the eventual problem evaluation and the relevance of the benchmark data to the computational technique. Examples of calculational methods are the three-dimensional multi-group or the pointwise cross section Monte Carlo codes, KENO-V.a or MCNP, respectively; the one-dimensional multi-group S_n discrete-ordinates transport theory code, ANISN; the diffusion theory code, GAMTEC II - HFN; and hand calculation methods such as the limited surface density, density analog, one- or two-group restricted three-dimensional diffusion theory, or solid angle methods. Associated computational data (e.g., cross section libraries, scattering quadrature sets, material bucklings, diffusion lengths, etc.) shall be identified. The computational method and associated computational data shall be described or referenced in the

1 validation documentation. When computer neutronics calculations are used, the type of computing
 2 platform should be stated along with relevant code configuration control information. This
 3 information may be provided by reference.

4
 5 An example partial listing of computer codes, models, and hand calculational methods that
 6 historically have been successfully used for nuclear criticality safety evaluations is provided in Table
 7 F.2.

8
 9 Table F.2. Partial Listing of Computer Codes, Models, and Hand Calculational Methods
 10

11 Computer Codes

- 14 - J. F. Briesmeister, Ed., "MCNP, A General Monte Carlo Code for Neutron and Photon Transport,"
 15 LA-7396-M, Rev. 2, Los Alamos National Lab. (Sept. 1986).
- 18 - L. L. Carter, C. R. Richey and C. E. Hughey, "GAMTEC-II: A Code for Generating Consistent
 19 Multigroup Constants Utilized in Diffusion and Transport Theory Calculations," BNWL-35,
 Pacific Northwest Laboratory, (March 1965).
- 21 - N. M. Greene, L. M. Petrie, "XSDRNP-S: A One-Dimensional Discrete-Ordinates Code for
 22 Transport Analysis," ORNL/NUREG/CSD-2/V2/R1 (June 1983).
- 24 - J. R. Lilley, "Computer Code HFN; Multi-Group, Multi-Region Neutron Diffusion Theory in One
 25 Space Dimension," HW-71545, General Electric Company, Richland, Washington, (July 1962).
- 27 - R. D. O'Dell, F. W. Brinkley, Jr., D. R. Marr, and R. E. Alcouffe, "Revised User's Manual for
 28 ONEDANT: A Code Package for One-Dimensional, Diffusion-Accelerated, Neutral-Particle
 29 Transport," LA-9184-M Rev. (December 1989). (On-line user's manuals for TWODANT are
 30 shipped with the program source.)
- 32 - L. Petrie, N. Landers, "KENO-Va, An Improved Monte Carlo Criticality Program with
 33 Supergrouping," ORNL/NUREG/CSD-2/VI/R2 (Dec. 1984).
- 35 - W. A. Rhoades and R. L. Childs, "An Updated Version of the DOT 4 One-and Two-Dimensional
 36 Neutron/Photon Transport Code," ORNL-5851 (July 1982).
- 38 - W. A. Rhoades and R. L. Childs, "The DORT Two-Dimensional Discrete Ordinates Transport Code,"
 39 *Nucl. Sci. Eng.* **99**, 1, 88-89 (May 1988).
- 41 - W. A. Rhoades and R. L. Childs, "The TORT Three-Dimensional Discrete Ordinates Neutron/Photon
 42 Transport Code," ORNL-6268 (November 1987).
- 44 - C. R. Richey, "EGGNIT: A Multigroup Cross Section Code," BNWL-1203, Battelle Memorial Institute
 45 Pacific Northwest Laboratories, Richland, Washington (November 1969).
- 47 - V. S. W. Sherriffs, "MONK, A General Purpose Monte Carlo Neutronics Program," SRD-R-86,
 48 United Kingdom Atomic Energy Authority Safety and Reliability Directorate, Culcheth
 49 Warrington, January 1978.

- 1 - T. P. Wilcox, E. M. Lent, "COG: A Particle Transport Code Designed to Solve the Boltzmann
2 Equation for Deep-Penetrating (Shielding) Problems." Draft Report, Lawrence Livermore
3 National Lab. (Oct. 1986).

4
5 Models and Hand Calculational Methods
6

- 7 - J.T. Thomas, "Solid Angle and Surface Density as Criticality Parameters," NUREG/CR-1615 and
8 ORNL/NUREG/CSD/TM-15, U.S. Nuclear Regulatory Commission (1980).
9
10 - J.T. Thomas, "Surface Density and Density Analog Models for Criticality in Arrays of Fissile
11 Materials," *Nucl. Sci. Eng.*, **62**, 424 (1977).
12
13 - M.C. Evans, "Criticality Assessment Using the Limiting Surface Density (NB^2n) Method and
14 Examples of Application," BNFL SAG/80/P29, British Nuclear Fuels plc (1980).
15
16 - H.F. Henry, C.E. Newlon and J.R. Knight, "Extensions of Neutron Interaction Criteria," K-1478
17 (July 1961).
18
19 - F.G. Welfare, "A Comparison of the Solid Angle Technique with KENO IV Calculations," *Trans. Am.*
20 *Nucl. Soc.*, **43**, 410 (1982).
21
22 - C.E. Newlon, "Solid Angle-Interaction Potential Method: Illustrative Problems," K-L-6328 (Sept.
23 1973).
24
25 - D.R. Oden, J.K. Thompson, M.A. Lewallen, "Critique of the Solid Angle Method," NUREG/CR-005,
26 U.S. Nuclear Regulatory Commission (1978).
27
28 - D. C. Hunt, "Comparative Calculational Evaluation of Array Criticality Models," *Nucl. Technol.*, **30**,
29 190 (1976).
30
31 - S. J. Altschuler and C. L. Schuske, "A Model for the Safe Storage of Fissile Solutions," *Nucl.*
32 *Technol.*, **17**, 110 (1973).
33
34 - C. L. Schuske and S. J. Altschuler, "Models for the Safe Storage of Dry and Wet Fissile Oxides,"
35 *Nucl. Technol.*, **19**, 84 (1973).
36
37 - S. J. Altschuler and C. L. Schuske, "Models for the Safe Storage of Fissile Metal," *Nucl. Technol.*,
38 **13**, 131 (1972).
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41 F.3 Description of similarities and differences. Nearly every computational model of a benchmark
42 experiment requires some modeling approximations. The computational model approximations of the
43 benchmarks shall be described and include discussions on the similarities and bases of the
44 differences.

45 F.4 Input variables. The geometric, material, and nuclear physics related input variables used for
46 the validation of the calculational or comparative method shall be provided along with sketches that
47 relate the benchmark to the computational model.

1 F.5 Acceptance criteria. The acceptance criteria are developed from the bias of calculated results
2 and the uncertainties of the experimental data, the calculational technique, and the calculational
3 models.

4
5 The basis for the calculational or comparative bias and the determination of acceptance criteria for
6 calculated subcritical results shall be provided. For nuclear criticality safety calculational method
7 validation purposes, the bias is defined as a measure of the systematic disagreement between the
8 results calculated by a method and experimental data. The usual method of determining the
9 calculational bias is to correlate the results of the benchmark critical experiments with the calculated
10 results of the code being validated. With a value of unity, $k_{eff} = 1.0$, for each benchmark critical
11 experiment, the bias is the deviation of the calculated values of k_{eff} from unity. The average bias is
12 usually determined by one of two methods: (1) taking the difference between a simple average of
13 the pooled calculated results and unity, that may be adequate for a specific validation, or (2) taking
14 the difference between a linear regression of the calculated results (as a function of some
15 independent variable, e.g., average energy group (AEG) of neutrons causing fission) and unity, that is
16 usually necessary for a global validation. The first method produces a single value for the bias, while
17 the second method produces a variable bias that is a function of the independent variable due to
18 trends. Generally, neither the bias nor its uncertainty is constant; both are typically a function of
19 one or more physical or nuclear variables. Physical variables include, for example, material
20 composition, density, and enrichment. Nuclear variables include AEG causing fission, ratio of
21 thermal absorption to total absorptions, ratio of total fissions to thermal fissions, fractional neutron
22 leakage, and others.

23
24 Uncertainties in the validation calculations come from three general sources. The first source arises
25 from limitations associated with the critical experiment and inadequacies of determinations and
26 documentation. These can include uncertainties in the material and fabrication tolerance of the
27 experimental hardware and fuel (compositions, assays, masses, densities, dimensions), the
28 experimenter's manipulation or adjustments, or both, to obtain the reported data, and an inadequate
29 description of the experimental layout and surroundings. The second source is from the
30 computational method, that may include uncertainties in the mathematical equations solved, the
31 calculational approximations utilized in solving the mathematical equations, the convergence criteria,
32 the cross-section data evaluation process and the manipulation of cross-section data, and limitations
33 of the computer hardware. The third source is from the calculational models developed to emulate
34 the experiment. These include uncertainties because of material and dimensional modeling
35 approximations, the selection of various code options, individual modeling/coding techniques, and
36 interpretation of the calculated results.

37
38 For computational method validation purposes, it is usually not practical or necessary to quantify and
39 qualify all the individual uncertainties. The total uncertainty can be estimated through the application
40 of any valid statistical treatment of the data. The total uncertainty determined usually appears as
41 the bias and a variability in the bias, depending upon the statistical analysis applied. The
42 combination of the bias and uncertainty in the bias is deduced from the mean value being calculated
43 to establish a subcritical value, e.g., acceptance criteria. This subcritical value and any other values
44 considered to be less subcritical are taken to be critical within the confidence limits applied to the
45 statistical technique to determine the uncertainty. A margin of subcriticality shall be deduced from
46 the previously described subcritical value to ensure subcriticality.

Where calculational methods of evaluation are used to predict neutron multiplication factors, the calculated multiplication factor, k_s , shall be equal to or less than an established allowable neutron multiplication factor, upper subcritical limit; i.e.,

$$k_s \leq k_c - \Delta k_s - \Delta k_c - \Delta k_m$$

where

k_s = the calculated allowable maximum multiplication factor, k_{eff} , upper subcritical limit (USL), of the system being evaluated for normal or credible abnormal conditions or events.

k_c = the mean k_{eff} that results from the calculation of the benchmark criticality experiments using a particular calculational method. If the calculated values of k_{eff} for the criticality experiments exhibit a trend with a physical or nuclear variable, then k_c shall be determined by extrapolation on the basis of a best fit to the calculated values. The criticality experiments used as benchmarks in computing k_c should have material compositions (neutron poisons and moderators), geometric configurations, neutron energy spectra, and nuclear characteristics (including reflectors) similar to those of the system being evaluated. Generally neither the bias nor its uncertainty is constant; both should be expected to be functions of composition and other variables.

Δk_s = an allowance for
(a) statistical or convergence uncertainties, or both, in the computation of k_s ,
(b) material and fabrication tolerances, and
(c) uncertainties due to limitations in the geometric or material representations used in the computational method.

Δk_c = a margin for uncertainty in k_c that includes allowance for
(a) uncertainties in the critical experiments,
(b) statistical or convergence uncertainties, or both, in the computation of k_c ,
(c) uncertainties due to extrapolation of k_c outside the range of experimental data, and
(d) uncertainties due to limitations in the geometrical or material representations used in the computational method.

Δk_m = an arbitrary margin to ensure the subcriticality of k_s . The margin in the correlating variable, that may be a function of composition and other variables, shall include allowances for the uncertainty in the bias and for uncertainties due to any extensions of the areas of applicability. A value for Δk_m should be described and documented, but in no case should a value of less than 0.02 be used.

F.6 Areas of applicability. An integral part of a code validation effort is to define the areas of applicability for the validation. There are three conditions that must be satisfied to ensure that calculations done to evaluate or support a real situation fall within the areas of applicability for the validation of the calculational method being used. These are materials (and associated nuclear properties), geometry, and neutron energy spectrum. Frequently, the correlating variable of AEG of a neutron causing fission is used to define an area of applicability for the validation and related computational bias. A discussion of the bases or judgments as to what constitutes the validation areas of applicability shall be provided.

The areas of applicability should identify the important variables and characteristics for which the code was (or was not) validated. For example, the areas of applicability may include specific types of fissionable materials (HEU, LEU, plutonium of low ^{240}Pu content, or others), material form (solution or metal, water-moderated or carbon-moderated, and others), geometric configurations (single units or arrays, heterogeneous or homogeneous, dissimilar units, or other conditions), and reflector materials (water, concrete, steel, lead, or others). The areas of applicability are intended to identify specific limits (upper and lower) of the variable or characteristic used to correlate the bias and uncertainties. For example, the areas of applicability may be defined in terms of the moderating ratio like $H:X = 10$ to 500, or in terms of the average energy group causing fission such as an AEG = 6.5 to 21.5, or in terms of the ratio of total fissions to thermal fissions like $F:F_{th} = 1.0$ to 5.0. For subsequent use of a validated code, the user should show that the variables and characteristics of the problem being calculated fall within the areas of applicability defined during the validation.

15 The areas of applicability of a calculational method may be extended beyond the range of
16 experimental conditions over which the bias is established by making use of correlated trends in the
17 bias. Where the extension is large, the method should be

- (a) validated with a stepwise approach in developing a repertoire of benchmarks for the purpose of identifying individual potentially compensating biases associated with individual changes in materials, geometries, or neutron spectra, and
 - (b) supplemented by other calculational methods to provide a better estimate of the bias(es) in the extended areas of applicability.

F.7 Example validation. This example describes a statistical technique used to establish the maximum allowable calculated k_{eff} , acceptance criterion (also called the upper subcritical limit) resulting from a computational method validation effort. Various elements of the technique are derived from different references. This example provides more detail than what is provided in the footnoted references, and the equations may be in a different, but algebraically identical, form. The equations in this example are usually in a basic form, while other algebraically identical forms (not presented here) are more convenient for computational purposes.

One method²⁵ used to validate the KENO criticality code and associated cross sections for establishing an acceptance criterion is to determine the single sided, uniform width, closed interval, lower tolerance band^{26,27} (LTB) for calculated k_{eff} values of critical systems. For application, this LTB becomes the upper subcritical limit (USL) acceptance criterion. A system is considered acceptably subcritical if a calculated k_{eff} plus two standard deviations lies below the USL or $k_{eff} + 2\sigma < \text{USL}$:

²⁵ H. R. Dyer et al., "A Technique for Code Validation for Criticality Safety Calculations," *Trans. Am. Nucl. Soc.* **63**, 238 (June 1991).

²⁶ D. C. Bowden and F. A. Graybill, "Confidence Bands of Uniform and Proportional Width for Linear Models," *Ann. Stat. Assoc. Jour.* **61**, 182 (March 1966).

²⁷ N. G. Johnson, Ed., "Tolerance Interval in Regression, Query 26," *Technometrics* 10, 107 (February 1968).

1 For a set of $n k_{eff}$ calculations of critical experiments with a corresponding independent variable x ,
 2 determine the linear least-squares fit, $k(x)$, of the data as a function of x .

3 $k(x) = b_0 + b_1 x$, where

$$b_1(\text{the slope}) = \frac{\sum(x_i - \bar{x})(k_i - \bar{k})}{\sum(x_i - \bar{x})^2},$$

$$b_0(\text{the intercept}) = \frac{\sum k_i - b_1 \sum x_i}{n},$$

$$\bar{x} = \frac{\sum x_i}{n},$$

5 and

$$\bar{k} = \frac{\sum k_i}{n}.$$

6 In these equations, and others to follow, a summation, Σ , means the sum of all values from $i=1$ to
 7 $i=n$, where n is the sample size, that is the number of critical experiments upon which the validation
 8 is based. The independent variable, x , is used to specify the areas of applicability, as described in
 9 section F.6.

10 The next step is to determine the "pooled" variance, s_p^2

$$s_p^2 = s_{k(x)}^2 + s_w^2, \text{where}$$

11 $s_{k(x)}^2$ (the variance of the fit, or mean square error) =

$$\frac{1}{n-2} \left[\sum(k_i - \bar{k})^2 - \frac{[\sum(x_i - \bar{x})(k_i - \bar{k})]^2}{\sum(x_i - \bar{x})^2} \right],$$

12 s_w^2 (the within variance of the data) = $\frac{1}{n} \sum \sigma_i^2$, and

13 σ_i is the standard deviation associated with each calculated k_{eff} .

14 The pooled standard deviation is then the square root of the variance, $s_p = \sqrt{s_p^2}$.

15 The within-variance, s_w^2 , represents the contribution of the variance from KENO or other Monte Carlo
 16 codes that have a standard deviation associated with the calculated k_{eff} values. For deterministic
 17 codes that do not have a standard deviation associated with the k_{eff} values, the within-variance is
 18 zero. It should be noted that the within-variance is not a part of the statistical method presented in
 19 footnotes 26 and 27, but was included here because of the inherent uncertainty from a Monte Carlo
 20 type code.

21 The next step is to determine a multiplier, C , of the pooled standard deviation such that there is at
 22 least α confidence that a proportion P of the population (of future calculations of critical systems)
 23 will lie above the line defined by $k(x)$ minus $C \cdot s_p$. This is the LTB as determined by the technique,
 24 and

25

1 $LTB = k(x) - C \cdot s_p .$

2 The α confidence, that is selected by the validator, is defined by

$$\alpha = 1 - \left(\frac{\gamma_1}{2} \right) - \gamma_2 , \text{ where}$$

4 $(1 - \gamma_1)$ = the one-sided confidence band about the linear regression, and

5 $(1 - \gamma_2)$ = the confidence on the variance of the fit.

6 Since the expression for α presents one equation and two unknowns, either γ_1 or γ_2 must be selected
 7 such that the other can be determined. In practice, the $(1 - \gamma_1)$ confidence is selected to be the
 8 same value as the α confidence, typically 0.95. With $\alpha = 0.95$ and $(1 - \gamma_1) = 0.95$, then
 9 $(1 - \gamma_2) = 0.975$. The proportion P is usually chosen to be 0.999.

10 The multiplier C is determined from

$$C = C^* + z_p \left[\frac{n-2}{\chi_{(n-2)(1-\gamma_2)}^2} \right]^{\frac{1}{2}} , \text{ where}$$

13 z_p is the standard normal variable of the proportion P for a normal distribution,

14 $\chi_{(n-2)(1-\gamma_2)}^2$ is from the Chi-square distribution for $(n-2)$ degrees of freedom at the $(1 - \gamma_2)$
 15 confidence,

16 and n is number of calculated critical experiments used in the validation.

17 NOTE: The author of footnote 27 is not consistent with the subscript notation used during the
 18 development of the technique. The technique is based upon the "upper tail" of the
 19 Chi-square distribution, such that

$$P[\chi_{(n-2)}^2 > \chi_{(n-2)(1-\gamma_2)}^2] = 1 - \gamma_2 .$$

20 Most Chi-square tables typically denote the "lower tail," such that

$$P[\chi_{(n-2)}^2 \leq \chi_{(n-2)(P)}^2] = P ;$$

21 then, the upper tail of the distribution is

$$P[\chi_{(n-2)}^2 > \chi_{(n-2)(P)}^2] = 1 - P .$$

22 Thus, to obtain the value of $\chi_{(n-2)(1-\gamma_2)}^2$ (author's notation), enter the table to find

23 $\chi_{(n-2)(\gamma_2)}^2$ (typical notation), as the author does in the example problem for footnote 27.

24 C^* is evaluated over the range of the independent variable, $a < x < b$, where a and b are,
 25 respectively, the lower and upper limits of the areas of applicability. C^* is determined by calculating
 26 values for g , h , ρ , and A , where

$$g = \left[\frac{1}{n} + \frac{(a - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right]^{\frac{1}{2}} ,$$

$$h = \left[\frac{1}{n} + \frac{(b - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right]^{\frac{1}{2}} ,$$

28 and

$$A = g/h .$$

$$\rho = \frac{1}{gh} \left[\frac{1}{n} + \frac{(a-\bar{x})(b-\bar{x})}{\sum(x_i-\bar{x})^2} \right],$$

1 The values of ρ , A , and $(n - 2)$ are used to determine a value D from Table F.7.1, at the $(1 - \gamma_1)$
2 confidence. Table F.7.1 covers the range of $0.5 \leq A \leq 1.5$; then

3 $C' = D \cdot g.$

4
5 If A is outside the range of 0.5 to 1.5, then use $1/A$, ρ , and $(n - 2)$ to determine D ; then

6 $C' = D \cdot h.$

7
8 In Table F.7.1, the values for D have been derived by evaluating the double integral given in footnote
9 26 and are essentially identical to the D values given in Table 3 of footnote 26. Table F.7.1 covers
10 the same range for $(n-2)$, ρ , A as the footnote, and also includes a 0.99 confidence (not included in
11 the footnoted table). Table F.7.1 is provided for users who may not have access to footnote 26 and
12 who may wish to impose a more restrictive confidence criteria.
13
14

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Table F.7.1. Calculated D Values

$(1 - \gamma_1) = 0.99$													
		A											
<u>(n-2)</u>	<u> ρ </u>	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	
4	.0	9.44	7.89	6.91	6.34	5.78	5.50	5.22	5.08	4.94	4.87	4.80	
4	.1	9.44	7.89	6.91	6.20	5.78	5.50	5.22	5.08	4.94	4.80	4.80	
4	.3	9.44	7.89	6.91	6.20	5.78	5.50	5.22	5.08	4.94	4.80	4.80	
4	.5	9.16	7.75	6.91	6.20	5.64	5.36	5.08	4.94	4.87	4.80	4.80	
4	.7	9.16	7.75	6.77	6.06	5.50	5.22	5.08	4.94	4.80	4.80	4.66	
4	.9	9.16	7.75	6.63	5.92	5.36	5.08	4.80	4.73	4.66	4.66	4.66	
6	.0	7.47	6.20	5.50	4.94	4.52	4.23	4.09	3.95	3.88	3.81	3.81	
6	.1	7.47	6.20	5.50	4.94	4.52	4.23	4.09	3.95	3.88	3.81	3.81	
6	.3	7.47	6.20	5.50	4.94	4.52	4.23	4.09	3.95	3.88	3.81	3.81	
6	.5	7.47	6.20	5.43	4.87	4.52	4.23	4.02	3.95	3.81	3.81	3.74	
6	.7	7.47	6.20	5.36	4.80	4.38	4.16	3.95	3.88	3.81	3.74	3.74	
6	.9	7.47	6.20	5.36	4.66	4.23	3.95	3.81	3.81	3.74	3.74	3.74	
8	.0	6.77	5.64	4.94	4.38	4.02	3.81	3.67	3.53	3.46	3.46	3.39	
8	.1	6.77	5.64	4.94	4.38	4.02	3.81	3.67	3.53	3.46	3.46	3.39	
8	.3	6.77	5.64	4.87	4.38	4.02	3.81	3.67	3.53	3.46	3.46	3.39	
8	.5	6.77	5.64	4.87	4.38	4.02	3.74	3.60	3.53	3.46	3.39	3.39	
8	.7	6.77	5.64	4.80	4.30	3.95	3.74	3.53	3.46	3.43	3.39	3.39	
8	.9	6.77	5.64	4.80	4.23	3.81	3.60	3.46	3.39	3.39	3.39	3.39	
10	.0	6.34	5.36	4.59	4.09	3.81	3.57	3.43	3.32	3.25	3.25	3.18	
10	.1	6.34	5.36	4.59	4.09	3.81	3.57	3.43	3.32	3.25	3.25	3.18	
10	.3	6.34	5.29	4.59	4.09	3.78	3.53	3.39	3.32	3.25	3.25	3.18	
10	.5	6.34	5.29	4.59	4.09	3.74	3.53	3.39	3.32	3.25	3.21	3.18	
10	.7	6.34	5.29	4.59	4.02	3.71	3.46	3.32	3.25	3.25	3.18	3.18	
10	.9	6.34	5.29	4.52	4.02	3.60	3.39	3.25	3.18	3.18	3.18	3.18	
12	.0	6.06	5.08	4.45	3.95	3.64	3.43	3.29	3.18	3.14	3.11	3.11	
12	.1	6.06	5.08	4.45	3.95	3.64	3.43	3.29	3.18	3.14	3.11	3.11	
12	.3	6.06	5.08	4.45	3.95	3.60	3.39	3.25	3.18	3.11	3.11	3.07	
12	.5	6.06	5.08	4.38	3.95	3.60	3.39	3.25	3.18	3.11	3.11	3.07	
12	.7	6.06	5.08	4.38	3.88	3.57	3.32	3.21	3.14	3.11	3.07	3.07	
12	.9	6.06	5.08	4.38	3.81	3.46	3.25	3.14	3.11	3.04	3.04	3.04	
14	.0	5.92	4.94	4.30	3.81	3.53	3.32	3.18	3.11	3.04	3.04	3.00	
14	.1	5.92	4.94	4.30	3.81	3.53	3.32	3.18	3.11	3.04	3.04	3.00	
14	.3	5.92	4.94	4.30	3.81	3.53	3.32	3.18	3.11	3.04	3.04	3.00	
14	.5	5.92	4.94	4.30	3.81	3.50	3.29	3.18	3.07	3.04	3.00	3.00	
14	.7	5.92	4.94	4.30	3.81	3.46	3.25	3.11	3.04	3.04	3.00	2.97	
14	.9	5.92	4.94	4.23	3.74	3.39	3.18	3.04	3.00	2.97	2.97	2.97	

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Table F.7.1 (cont.)

$(1 - \gamma_1) = 0.99$ (cont.)												
A												
$(n-2)$	ρ	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
16	.0	5.85	4.87	4.23	3.74	3.46	3.25	3.11	3.04	2.97	2.97	2.93
16	.1	5.85	4.87	4.23	3.74	3.46	3.25	3.11	3.04	2.97	2.97	2.93
16	.3	5.85	4.87	4.23	3.74	3.46	3.25	3.11	3.04	2.97	2.97	2.93
16	.5	5.85	4.87	4.23	3.74	3.43	3.21	3.11	3.04	2.97	2.97	2.93
16	.7	5.85	4.87	4.16	3.74	3.39	3.18	3.04	3.00	2.97	2.93	2.93
16	.9	5.85	4.87	4.16	3.67	3.32	3.11	3.00	2.97	2.93	2.93	2.90
20	.0	5.71	4.73	4.09	3.67	3.36	3.14	3.04	2.93	2.90	2.86	2.86
20	.1	5.71	4.73	4.09	3.67	3.36	3.14	3.04	2.93	2.90	2.86	2.86
20	.3	5.71	4.73	4.09	3.64	3.32	3.14	3.00	2.93	2.90	2.86	2.86
20	.5	5.71	4.73	4.09	3.64	3.32	3.11	3.00	2.93	2.90	2.86	2.86
20	.7	5.71	4.73	4.09	3.60	3.29	3.11	2.97	2.90	2.86	2.86	2.86
20	.9	5.71	4.73	4.09	3.57	3.25	3.04	2.90	2.86	2.86	2.83	2.83
24	.0	5.57	4.66	4.02	3.60	3.29	3.07	2.97	2.90	2.83	2.83	2.83
24	.1	5.57	4.66	4.02	3.60	3.29	3.07	2.97	2.90	2.83	2.83	2.83
24	.3	5.57	4.66	4.02	3.60	3.29	3.07	2.97	2.90	2.83	2.83	2.83
24	.5	5.57	4.66	4.02	3.57	3.25	3.07	2.93	2.86	2.83	2.83	2.79
24	.7	5.57	4.66	4.02	3.53	3.25	3.04	2.93	2.86	2.83	2.83	2.79
24	.9	5.57	4.66	4.02	3.53	3.18	2.97	2.86	2.83	2.79	2.79	2.79
30	.0	5.50	4.59	3.95	3.53	3.21	3.04	2.90	2.83	2.79	2.76	2.76
30	.1	5.50	4.59	3.95	3.53	3.21	3.04	2.90	2.83	2.79	2.76	2.76
30	.3	5.50	4.59	3.95	3.53	3.21	3.04	2.90	2.83	2.79	2.76	2.76
30	.5	5.50	4.59	3.95	3.50	3.21	3.00	2.90	2.83	2.79	2.76	2.76
30	.7	5.50	4.59	3.95	3.46	3.18	2.97	2.86	2.79	2.76	2.76	2.76
30	.9	5.50	4.59	3.95	3.46	3.11	2.90	2.83	2.76	2.76	2.76	2.76
40	.0	5.43	4.52	3.88	3.46	3.14	2.97	2.86	2.79	2.76	2.72	2.72
40	.1	5.43	4.52	3.88	3.46	3.14	2.97	2.86	2.79	2.76	2.72	2.72
40	.3	5.43	4.52	3.88	3.46	3.14	2.97	2.86	2.79	2.76	2.72	2.72
40	.5	5.43	4.52	3.88	3.46	3.14	2.97	2.83	2.76	2.72	2.72	2.72
40	.7	5.43	4.52	3.88	3.43	3.11	2.93	2.83	2.76	2.72	2.72	2.72
40	.9	5.43	4.52	3.88	3.39	3.04	2.86	2.76	2.72	2.72	2.69	2.69
50	.0	5.36	4.45	3.85	3.39	3.11	2.93	2.83	2.76	2.72	2.69	2.69
50	.1	5.36	4.45	3.85	3.39	3.11	2.93	2.83	2.76	2.72	2.69	2.69
50	.3	5.36	4.45	3.85	3.39	3.11	2.93	2.83	2.76	2.72	2.69	2.69
50	.5	5.36	4.45	3.85	3.39	3.11	2.93	2.79	2.76	2.72	2.69	2.69
50	.7	5.36	4.45	3.81	3.39	3.07	2.90	2.79	2.72	2.69	2.69	2.69
50	.9	5.36	4.45	3.81	3.36	3.04	2.83	2.72	2.69	2.69	2.69	2.69

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Table F.7.1 (cont.)

		$(1 - \gamma_1) = 0.95$											
		A											
<u>(n-2)</u>	<u> ρ </u>	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	
4	.0	5.68	4.81	4.25	3.85	3.58	3.39	3.23	3.13	3.04	2.99	2.93	
4	.1	5.68	4.80	4.23	3.85	3.58	3.37	3.23	3.13	3.04	2.99	2.93	
4	.3	5.64	4.80	4.23	3.83	3.57	3.36	3.21	3.11	3.04	2.97	2.91	
4	.5	5.64	4.76	4.18	3.78	3.51	3.32	3.16	3.07	2.99	2.93	2.90	
4	.7	5.61	4.71	4.11	3.71	3.43	3.23	3.09	3.00	2.93	2.88	2.86	
4	.9	5.57	4.66	4.01	3.57	3.25	3.07	2.95	2.88	2.83	2.81	2.79	
6	.0	4.94	4.18	3.67	3.34	3.09	2.92	2.79	2.71	2.63	2.58	2.55	
6	.1	4.94	4.18	3.67	3.34	3.09	2.92	2.79	2.71	2.63	2.58	2.55	
6	.3	4.94	4.16	3.67	3.32	3.07	2.90	2.78	2.69	2.62	2.57	2.55	
6	.5	4.94	4.15	3.64	3.29	3.04	2.86	2.74	2.65	2.60	2.55	2.53	
6	.7	4.90	4.13	3.58	3.21	2.97	2.79	2.69	2.60	2.55	2.51	2.49	
6	.9	4.90	4.09	3.51	3.13	2.85	2.67	2.57	2.51	2.48	2.46	2.46	
8	.0	4.64	3.92	3.44	3.11	2.88	2.72	2.60	2.52	2.46	2.42	2.39	
8	.1	4.64	3.92	3.44	3.11	2.88	2.72	2.60	2.52	2.46	2.42	2.39	
8	.3	4.64	3.92	3.43	3.09	2.86	2.71	2.59	2.51	2.45	2.41	2.38	
8	.5	4.62	3.90	3.41	3.07	2.83	2.67	2.56	2.48	2.42	2.39	2.36	
8	.7	4.62	3.87	3.36	3.02	2.78	2.62	2.51	2.44	2.39	2.36	2.34	
8	.9	4.62	3.85	3.30	2.93	2.67	2.51	2.42	2.36	2.34	2.32	2.31	
10	.0	4.48	3.78	3.30	2.99	2.77	2.61	2.49	2.42	2.36	2.32	2.29	
10	.1	4.48	3.78	3.30	2.99	2.76	2.61	2.49	2.42	2.36	2.32	2.29	
10	.3	4.48	3.76	3.29	2.97	2.75	2.60	2.49	2.41	2.35	2.32	2.28	
10	.5	4.46	3.76	3.27	2.95	2.72	2.56	2.46	2.39	2.34	2.30	2.27	
10	.7	4.46	3.74	3.23	2.90	2.67	2.52	2.42	2.34	2.30	2.27	2.26	
10	.9	4.45	3.71	3.20	2.83	2.58	2.42	2.33	2.27	2.25	2.24	2.23	
12	.0	4.38	3.69	3.21	2.91	2.69	2.54	2.43	2.35	2.30	2.27	2.24	
12	.1	4.38	3.68	3.21	2.91	2.69	2.54	2.43	2.35	2.30	2.27	2.24	
12	.3	4.38	3.67	3.21	2.90	2.68	2.53	2.42	2.34	2.29	2.26	2.23	
12	.5	4.38	3.67	3.20	2.87	2.65	2.50	2.40	2.33	2.27	2.24	2.22	
12	.7	4.36	3.65	3.16	2.83	2.61	2.46	2.35	2.29	2.25	2.22	2.20	
12	.9	4.36	3.64	3.13	2.76	2.51	2.36	2.27	2.22	2.20	2.19	2.19	
14	.0	4.30	3.62	3.16	2.85	2.64	2.49	2.39	2.31	2.26	2.22	2.20	
14	.1	4.30	3.62	3.16	2.85	2.64	2.49	2.39	2.31	2.26	2.22	2.20	
14	.3	4.30	3.62	3.15	2.85	2.63	2.48	2.38	2.30	2.25	2.21	2.20	
14	.5	4.30	3.60	3.14	2.82	2.61	2.46	2.35	2.28	2.23	2.20	2.18	
14	.7	4.29	3.58	3.11	2.78	2.56	2.42	2.32	2.25	2.20	2.18	2.17	
14	.9	4.29	3.58	3.07	2.71	2.48	2.32	2.23	2.19	2.16	2.15	2.15	

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Table F.7.1 (cont.)

$(1 - \gamma_1) = 0.95$ (cont.)												
A												
<u>(n-2)</u>	<u> ρ </u>	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
16	.0	4.25	3.57	3.13	2.81	2.61	2.46	2.35	2.28	2.23	2.20	2.17
16	.1	4.25	3.57	3.12	2.81	2.60	2.46	2.35	2.28	2.23	2.20	2.17
16	.3	4.25	3.57	3.11	2.80	2.59	2.45	2.34	2.27	2.22	2.19	2.16
16	.5	4.25	3.56	3.09	2.78	2.57	2.42	2.32	2.26	2.20	2.18	2.15
16	.7	4.23	3.55	3.07	2.75	2.53	2.38	2.28	2.22	2.18	2.15	2.14
16	.9	4.23	3.53	3.04	2.69	2.44	2.29	2.20	2.16	2.13	2.13	2.13
20	.0	4.18	3.51	3.07	2.76	2.56	2.42	2.31	2.24	2.19	2.15	2.13
20	.1	4.18	3.51	3.06	2.76	2.56	2.41	2.31	2.24	2.19	2.15	2.13
20	.3	4.18	3.50	3.06	2.75	2.55	2.40	2.30	2.23	2.18	2.14	2.13
20	.5	4.18	3.50	3.04	2.73	2.52	2.38	2.28	2.21	2.17	2.13	2.12
20	.7	4.16	3.49	3.02	2.70	2.49	2.34	2.24	2.18	2.14	2.12	2.11
20	.9	4.16	3.48	2.99	2.63	2.41	2.26	2.17	2.13	2.10	2.09	2.09
24	.0	4.13	3.47	3.02	2.72	2.52	2.38	2.28	2.21	2.16	2.13	2.11
24	.1	4.13	3.47	3.02	2.72	2.52	2.38	2.28	2.20	2.16	2.13	2.10
24	.3	4.13	3.46	3.02	2.72	2.51	2.37	2.27	2.20	2.15	2.12	2.10
24	.5	4.13	3.46	3.00	2.71	2.49	2.35	2.25	2.19	2.14	2.11	2.09
24	.7	4.13	3.44	2.99	2.67	2.45	2.31	2.21	2.15	2.12	2.09	2.08
24	.9	4.13	3.44	2.95	2.61	2.37	2.23	2.14	2.10	2.08	2.07	2.06
30	.0	4.09	3.43	2.99	2.70	2.49	2.35	2.25	2.18	2.13	2.10	2.08
30	.1	4.09	3.43	2.99	2.70	2.49	2.35	2.25	2.18	2.13	2.10	2.08
30	.3	4.09	3.43	2.99	2.69	2.49	2.34	2.24	2.18	2.13	2.10	2.07
30	.5	4.09	3.43	2.97	2.67	2.46	2.32	2.22	2.16	2.12	2.09	2.06
30	.7	4.09	3.41	2.95	2.63	2.42	2.28	2.19	2.13	2.09	2.07	2.05
30	.9	4.08	3.41	2.92	2.58	2.34	2.20	2.12	2.07	2.05	2.05	2.05
40	.0	4.04	3.39	2.95	2.66	2.46	2.32	2.22	2.15	2.11	2.07	2.05
40	.1	4.04	3.39	2.95	2.66	2.46	2.32	2.22	2.15	2.11	2.07	2.05
40	.3	4.04	3.39	2.95	2.65	2.45	2.31	2.21	2.15	2.10	2.07	2.05
40	.5	4.04	3.38	2.93	2.63	2.43	2.29	2.20	2.13	2.09	2.06	2.05
40	.7	4.04	3.37	2.92	2.61	2.40	2.26	2.16	2.11	2.07	2.05	2.04
40	.9	4.04	3.37	2.89	2.55	2.32	2.18	2.10	2.05	2.04	2.03	2.02
50	.0	4.02	3.37	2.93	2.64	2.44	2.30	2.20	2.14	2.09	2.06	2.04
50	.1	4.02	3.37	2.93	2.64	2.44	2.30	2.20	2.14	2.09	2.06	2.04
50	.3	4.02	3.36	2.92	2.63	2.43	2.29	2.20	2.13	2.09	2.05	2.04
50	.5	4.02	3.36	2.92	2.62	2.42	2.27	2.18	2.12	2.07	2.05	2.03
50	.7	4.02	3.36	2.90	2.59	2.38	2.24	2.15	2.09	2.05	2.03	2.02
50	.9	4.02	3.35	2.87	2.54	2.31	2.16	2.08	2.04	2.02	2.01	2.01

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Table F.7.1 (cont.)

$(1 - \gamma_1) = 0.90$												
A												
$(n-2)$	ρ_1	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
4	.0	4.38	3.74	3.32	3.03	2.82	2.66	2.55	2.46	2.39	2.34	2.30
4	.1	4.38	3.74	3.32	3.02	2.81	2.66	2.55	2.46	2.39	2.34	2.30
4	.3	4.38	3.72	3.30	3.00	2.79	2.64	2.53	2.44	2.37	2.32	2.28
4	.5	4.34	3.69	3.26	2.96	2.75	2.60	2.49	2.41	2.34	2.29	2.26
4	.7	4.30	3.64	3.20	2.89	2.67	2.52	2.42	2.34	2.28	2.25	2.21
4	.9	4.27	3.58	3.09	2.76	2.53	2.38	2.28	2.23	2.19	2.17	2.16
6	.0	3.95	3.37	2.99	2.71	2.52	2.38	2.28	2.20	2.14	2.10	2.06
6	.1	3.95	3.37	2.98	2.71	2.52	2.38	2.28	2.20	2.14	2.10	2.06
6	.3	3.94	3.36	2.97	2.70	2.50	2.37	2.27	2.19	2.13	2.09	2.05
6	.5	3.94	3.33	2.93	2.66	2.47	2.33	2.23	2.16	2.10	2.06	2.03
6	.7	3.91	3.29	2.88	2.60	2.41	2.27	2.18	2.11	2.06	2.02	2.00
6	.9	3.88	3.25	2.80	2.49	2.29	2.15	2.07	2.02	1.98	1.97	1.96
8	.0	3.77	3.21	2.83	2.57	2.40	2.26	2.16	2.09	2.04	1.99	1.96
8	.1	3.77	3.21	2.83	2.57	2.39	2.26	2.16	2.09	2.03	1.99	1.96
8	.3	3.76	3.19	2.82	2.56	2.38	2.25	2.15	2.08	2.02	1.98	1.95
8	.5	3.75	3.17	2.79	2.53	2.34	2.21	2.12	2.05	2.00	1.96	1.93
8	.7	3.73	3.14	2.74	2.48	2.29	2.16	2.07	2.00	1.96	1.93	1.91
8	.9	3.72	3.11	2.68	2.38	2.18	2.05	1.97	1.92	1.90	1.88	1.87
10	.0	3.66	3.11	2.75	2.50	2.32	2.20	2.10	2.03	1.98	1.93	1.90
10	.1	3.66	3.11	2.75	2.49	2.32	2.19	2.10	2.02	1.97	1.93	1.90
10	.3	3.65	3.10	2.73	2.49	2.31	2.18	2.09	2.02	1.96	1.92	1.89
10	.5	3.65	3.08	2.71	2.45	2.27	2.15	2.05	1.99	1.94	1.91	1.88
10	.7	3.64	3.06	2.67	2.41	2.22	2.10	2.01	1.95	1.91	1.87	1.85
10	.9	3.63	3.02	2.61	2.32	2.13	2.00	1.92	1.87	1.84	1.83	1.82
12	.0	3.60	3.05	2.69	2.45	2.27	2.15	2.05	1.98	1.93	1.90	1.87
12	.1	3.60	3.05	2.69	2.45	2.27	2.15	2.05	1.98	1.93	1.89	1.86
12	.3	3.59	3.04	2.68	2.43	2.26	2.13	2.04	1.98	1.92	1.88	1.86
12	.5	3.58	3.02	2.66	2.41	2.23	2.11	2.02	1.95	1.90	1.87	1.84
12	.7	3.58	3.00	2.62	2.36	2.18	2.06	1.97	1.91	1.87	1.84	1.82
12	.9	3.57	2.98	2.56	2.27	2.09	1.96	1.88	1.83	1.81	1.80	1.79
14	.0	3.55	3.01	2.65	2.42	2.24	2.12	2.03	1.96	1.91	1.87	1.84
14	.1	3.55	3.01	2.65	2.41	2.24	2.12	2.02	1.96	1.91	1.87	1.84
14	.3	3.55	3.00	2.64	2.40	2.23	2.10	2.02	1.94	1.90	1.86	1.83
14	.5	3.54	2.99	2.62	2.37	2.20	2.08	1.99	1.92	1.87	1.84	1.82
14	.7	3.53	2.96	2.58	2.33	2.15	2.03	1.94	1.88	1.84	1.81	1.80
14	.9	3.52	2.94	2.53	2.25	2.06	1.94	1.86	1.81	1.79	1.77	1.77

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Table F.7.1 (cont.)

$(1 - \gamma_1) = 0.90$ (cont.)												
A												
<u>(n-2)</u>	<u> ρ </u>	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
16	.0	3.51	2.98	2.63	2.39	2.22	2.09	2.01	1.94	1.89	1.85	1.82
16	.1	3.51	2.98	2.63	2.39	2.22	2.09	2.00	1.94	1.88	1.85	1.82
16	.3	3.51	2.97	2.62	2.38	2.20	2.08	1.99	1.93	1.87	1.84	1.81
16	.5	3.50	2.96	2.59	2.35	2.18	2.05	1.97	1.91	1.86	1.82	1.80
16	.7	3.50	2.93	2.56	2.31	2.13	2.01	1.92	1.87	1.83	1.80	1.78
16	.9	3.50	2.92	2.51	2.23	2.04	1.92	1.84	1.80	1.77	1.76	1.75
20	.0	3.47	2.94	2.59	2.35	2.19	2.06	1.98	1.91	1.86	1.82	1.80
20	.1	3.47	2.93	2.59	2.35	2.19	2.06	1.98	1.91	1.86	1.82	1.79
20	.3	3.47	2.93	2.58	2.34	2.17	2.05	1.96	1.90	1.85	1.81	1.79
20	.5	3.46	2.92	2.56	2.31	2.15	2.03	1.94	1.88	1.83	1.80	1.77
20	.7	3.45	2.90	2.52	2.27	2.10	1.98	1.90	1.84	1.80	1.77	1.76
20	.9	3.45	2.88	2.48	2.20	2.01	1.89	1.82	1.77	1.75	1.73	1.71
24	.0	3.44	2.91	2.56	2.33	2.16	2.05	1.96	1.89	1.84	1.80	1.78
24	.1	3.44	2.91	2.56	2.33	2.16	2.05	1.95	1.89	1.84	1.80	1.78
24	.3	3.43	2.90	2.56	2.32	2.15	2.03	1.94	1.88	1.83	1.80	1.77
24	.5	3.43	2.89	2.53	2.29	2.13	2.01	1.92	1.86	1.81	1.78	1.76
24	.7	3.43	2.87	2.50	2.25	2.08	1.96	1.88	1.82	1.78	1.76	1.74
24	.9	3.43	2.85	2.46	2.18	1.99	1.87	1.80	1.76	1.73	1.72	1.72
30	.0	3.41	2.88	2.54	2.31	2.14	2.02	1.94	1.87	1.82	1.79	1.76
30	.1	3.41	2.88	2.54	2.31	2.14	2.02	1.94	1.87	1.82	1.79	1.76
30	.3	3.41	2.88	2.53	2.30	2.13	2.01	1.93	1.86	1.81	1.78	1.75
30	.5	3.40	2.86	2.51	2.27	2.11	1.99	1.91	1.84	1.80	1.76	1.74
30	.7	3.40	2.85	2.48	2.23	2.06	1.94	1.87	1.81	1.77	1.74	1.73
30	.9	3.39	2.83	2.43	2.16	1.98	1.86	1.79	1.74	1.72	1.71	1.70
40	.0	3.38	2.85	2.52	2.28	2.13	2.01	1.92	1.85	1.80	1.77	1.74
40	.1	3.38	2.85	2.52	2.28	2.12	2.01	1.92	1.85	1.80	1.77	1.74
40	.3	3.38	2.85	2.51	2.27	2.11	1.99	1.91	1.84	1.80	1.76	1.74
40	.5	3.37	2.84	2.49	2.25	2.09	1.97	1.89	1.83	1.78	1.75	1.73
40	.7	3.37	2.82	2.46	2.21	2.05	1.93	1.85	1.79	1.75	1.73	1.71
40	.9	3.36	2.81	2.42	2.14	1.96	1.84	1.77	1.73	1.70	1.69	1.69
50	.0	3.36	2.85	2.50	2.27	2.11	1.99	1.91	1.84	1.80	1.76	1.73
50	.1	3.36	2.84	2.50	2.27	2.11	1.99	1.91	1.84	1.80	1.76	1.73
50	.3	3.36	2.84	2.49	2.26	2.10	1.98	1.90	1.83	1.79	1.75	1.73
50	.5	3.36	2.83	2.48	2.24	2.08	1.96	1.87	1.81	1.77	1.74	1.72
50	.7	3.36	2.81	2.45	2.20	2.03	1.92	1.84	1.78	1.74	1.72	1.70
50	.9	3.36	2.79	2.41	2.13	1.95	1.83	1.76	1.72	1.69	1.69	1.68

Once these values have been determined, the linear regression, $k(x)$, and the LTB should be graphically depicted for future reference. The LTB is the USL for the maximum allowable k_{eff} , k_s , as a function of the independent variable, as shown in Figure F.7.1. For application, a calculated k_{eff} plus two standard deviations shall lie below the USL line, $k_{eff}, k_s + 2\sigma < \text{USL}$.

This statistical method for code validation allows the USL to be established such that there is a high degree of confidence that a calculated result that satisfies the acceptance criteria is indeed subcritical. Although a margin of subcriticality is not determined by the technique, a margin can be defined as the difference between the $(1 - \gamma_1)$ confidence on the linear regression for a single future calculation and the USL. The $(1 - \gamma_1)$ confidence on a single future calculation is determined by

$$w(x) = t_{(1-\gamma_1)} s_p \left[1 + \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right]^{\frac{1}{2}}, \text{ where}$$

$t_{(1-\gamma_1)}$ is from the student-t distribution at the $(1 - \gamma_1)$ confidence and $(n - 2)$ degrees of freedom, and

s_p is the pooled standard deviation previously determined.

Since $w(x)$ is a curvilinear function, and it is desirable to have a constant width margin, the expression is evaluated at $x_i = a$ and $x_i = b$ (the lowest and highest values, respectively, of the independent variable). The larger of the two is the constant W , to be deducted from the linear regression, $k(x)$, to provide a uniform width confidence band for a single future calculation. The margin of subcriticality is the difference between the uniform width confidence band for a single future calculation and the USL, or

$$\begin{aligned} \text{margin of subcriticality} &= [k(x) - W] - [k(x) - C \cdot s_p] \\ &= C \cdot s_p - W. \end{aligned}$$

Figure F.7.1 graphically depicts typical results of the single-sided, uniform-width, closed-interval, LTB technique for code validation. Since the calculational bias has been accounted for in the linear regression, it is not uniquely determined. Numerically, the average calculational bias at any point within the areas of applicability is the difference between the linear regression $k(x)$ and unity.

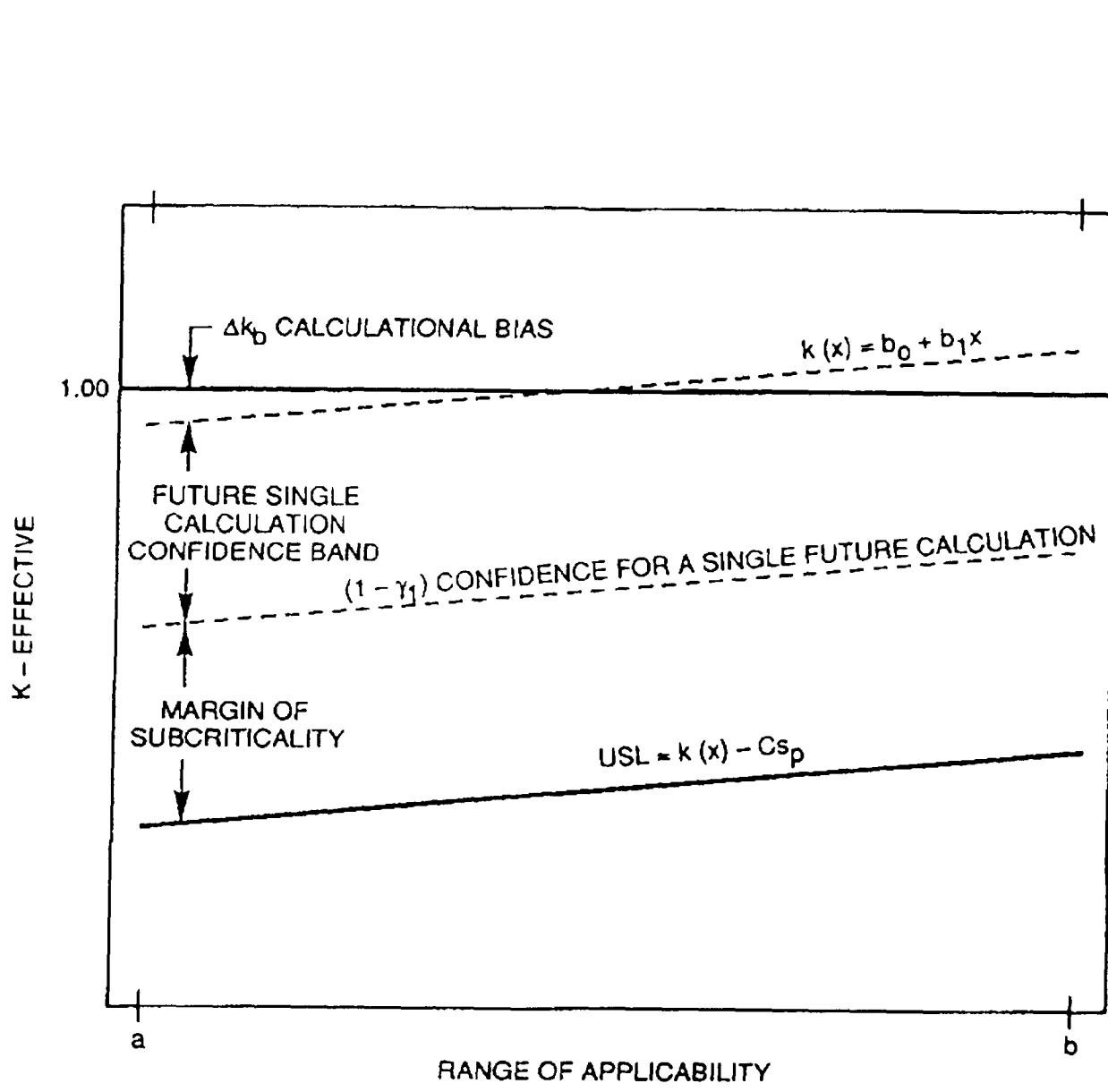


Figure F.7.1. Typical results for the single-sided, uniform-width, closed-interval, LTB technique.

1 As an example of this technique, assume that 29 critical experiments have been modeled and
 2 calculated. The calculated k_{eff} , standard deviation, and average energy group causing fission (the
 3 independent variable, x) are shown in Table F.7.2.

5 Table F.7.2. Input Data for Example Problem

	k_{eff}	σ	AEG
7	0.99647	0.00337	14.82
8	0.99776	0.00326	14.81
9	1.00764	0.00311	14.83
10	0.99587	0.00365	14.44
11	0.99744	0.00327	14.28
12	1.00337	0.00335	14.84
13	0.99609	0.00395	14.73
14	1.00108	0.00378	15.08
15	0.99737	0.00325	15.20
16	0.98408	0.00342	15.31
17	0.98871	0.00361	15.26
18	0.99527	0.00292	15.50
19	0.98804	0.00273	15.49
20	1.01363	0.00401	14.36
21	1.01660	0.00445	14.36
22	1.00874	0.00485	14.36
23	1.01190	0.00479	14.38
24	1.00980	0.00498	14.35
25	1.00565	0.00397	14.10
26	1.01929	0.00407	14.12
27	1.00860	0.00411	14.10
28	0.99487	0.00462	15.04
29	0.99257	0.00382	14.90
30	1.00132	0.00450	14.90
31	0.99154	0.00420	14.90
32	1.00028	0.00374	15.43
33	0.99565	0.00413	15.44
34	0.98574	0.00415	15.43
35	0.98733	0.00416	15.43

36
 37
 38
 39 Table F.7.3 summarizes the various terms calculated to establish the USL and margin of
 40 subcriticality, with $\rho = 0.999$ and $\sigma = 0.95$. From these results, the USL is defined by the straight
 41 line $USL = 1.1900 - 0.0153 x$, and is statistically valid only between the range of AEG from 14.10
 42 to 15.50. Any calculated $k_{eff} + 2\sigma$ that is below the USL is adequately subcritical, with a margin of
 43 subcriticality of at least 0.02.
 44

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Table F.7.3. Calculated Terms for Example Problem

1		
2	<i>n</i>	= 29
3	linear regression, $k(x)$	= $1.2266 - 0.015295 x$
4	minimum value of x , a	= 14.10
5	maximum value of x , b	= 15.50
6	average x (AEG), \bar{x}	= 14.8341
7	average k_{eff} , \bar{k}	= 0.99975
8	variance of fit, $s_{k,x}^2$	= 3.8260-05*
9	within variance, s_w^2	= 1.5304-05
10	pooled variance, s_p^2	= 5.3564-05
11	pooled standard deviation, s_p	= 7.3187-03
12	$Z_p @ P = 0.999$	= 3.090
13	$x^* @ (n - 2), (1 - \gamma_1)$	= 14.57
14	g	= 0.3497
15	h	= 0.3266
16	ρ	= -0.3951
17	A	= 1.0705
18	D	= 2.274 (interpolated from Table C-1)
19	C'	= 0.79525
20	C	= 4.9973
21	$C \cdot s_p$	= 0.0366
22	LTB = USL	= $1.1900 - 0.015295 x$
23	student-t @ $(n - 2) (1 - \gamma_1)$	= 1.703
24	W (max. at $x = a$ and $x = b$)	= 0.0132
25		
26	minimum margin of	= 0.0234
27	subcriticality, $C \cdot s_p \cdot w$	

*Read as 3.8260×10^{-5} .

Figure F.7.2 provides a plot of the resultant single-sided, uniform-width, closed-interval, lower tolerance band technique developed from the example.

There may be valid reasons to reduce the USL. There are many factors that may both change the AEG and affect other parameters as well in a multiplying manner. For example, suppose very low temperatures cannot be ruled out for the application in question. Low temperatures will increase the AEG and can also increase density. Therefore cold increases reactivity by increasing density, may change the AEG to be outside the range of applicability of critical experiments, and decreases the margin of safety.

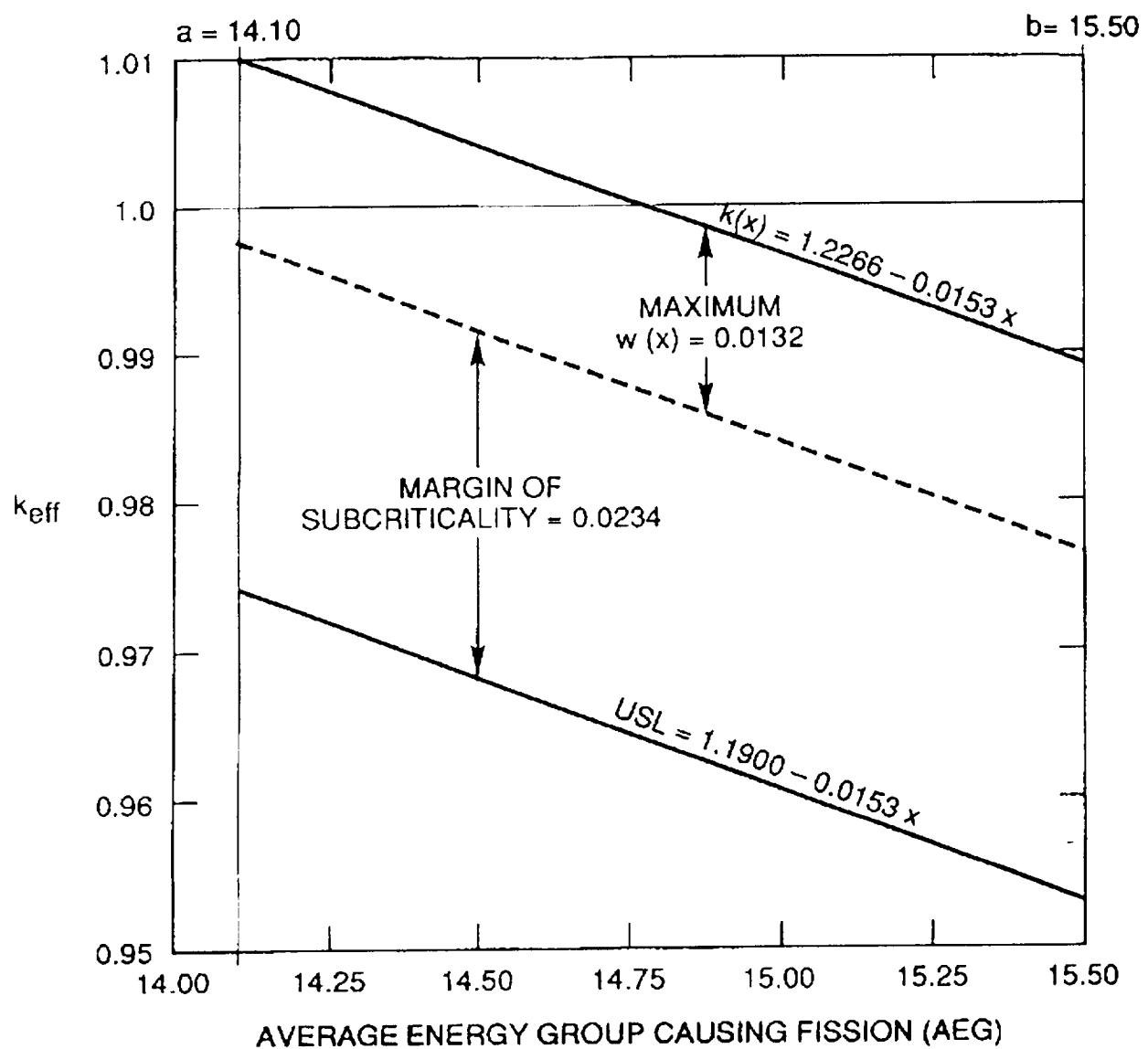


Figure F.7.2. Example results for the single-sided, uniform-width, closed-interval, LTB technique.